Gedanken experiments for the identification of microstructured continua

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In this lecture we will analyze the role of the principle of virtual work in continuum mechanics and an introduction of first and second gradient continua will be therefore given. Within this context, material characterization is done in terms of the functional dependence of the internal energy and with the identification of all its coefficients. Characterization of first gradient elastic continua is standard in the literature. Recently, the same problem for second gradient materials has been investigated. Even though it is not complete, a dynamic investigation is done with an analysis of the dispersion relation. Another possibility is a static characterization, that is achieved through proper ideal (or gedanken) experiments.

In particular, we will consider a two-dimensional solid consisting of a linear elastic isotropic material, for which the deformation (strain or internal) energy depends on the second gradient of the displacement and it is demonstrated to depend on 6 constitutive parameters: the 2 Lamé constants and 4 more parameters. Therefore, gedanken experiments are designed to allow to write equations that have as unknowns the six constants that define the internal energy and as known terms the values of the experimental measurements of appropriately selected quantities.

Besides, we relax the isotropic hypothesis and consider a material that is invariant under 90 degrees rotation and invariant for mirror transformation. The reason is that such anisotropy is the most general for pantographic structures that are composed by two identical orthogonal families of bars interconnected with ideal pivots. It is well known in the literature that the corresponding strain energy depends on 8 constitutive parameters: 3 parameters related to the first gradient part and 5 parameters related to the second gradient part. We finally show that, with this procedure, the whole set of the 8 constitutive parameters are completely characterized in terms of the materials of which the bars are built, of their cross sections and of the distance between the nearest pivots. On the basis of these considerations, a remarkable form of the strain energy is derived in terms of the displacement fields, that closely resembles the strain energy of simple Euler beams. Numerical simulations confirm the validity of the presented results, classic bone-shaped deformations are derived in classic bias numerical tests and the presence of floppy mode is also made numerically evident in the present continuum model.