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LETTERS TO THE EDITOR

ON LARGE-AMPLITUDE VIBRATIONS OF CABLES

A paper by Al-Noury and Ali entitled "Large-amplitude vibrations of parabolic cables" was recently published in this Journal [1]. It appeared to us that their aim was to refocus on the main aspects of the non-linear dynamics of cables which have been the subject of several recent works and to re-examine the procedures most frequently used to handle the spatial and temporal dependence. Although in reference [1] forced coupled in-plane and out-of-plane oscillations were studied, a problem not previously analyzed, the authors made no mention of a certain number of earlier contributions [2–7] which might have helped in their study of this problem.

A suspended cable exhibits a dynamic behaviour different from that of a taut string. This is due to the curvature of the initial configuration which leads to quadratic non-linearities. The presence of both quadratic and cubic terms in the equations of motion produce softening or hardening behaviour depending on the geometrical and mechanical characteristics, while a taut string shows only hardening behaviour associated with cubic non-linearities. This phenomenon which is peculiar to cables remains almost completely in the shadow in reference [1]. In particular, within the range of small amplitudes, where the behaviour is well represented by an asymptotic expansion up to the second order, the frequency–amplitude curves (which are the backbone of the frequency-response curves) can be softening or hardening both in the case of in-plane and out-of-plane oscillations [5]. For larger amplitudes the frequency–amplitude curves of in-plane oscillations can show a point of contra-flexion, as is shown in reference [6]: that is, one cable can exhibit softening response in the low amplitude range, and thereafter hardening response.

The linearized symmetric free oscillations of a cable in planar motion were shown [8] to depend purely on the mechanical/geometrical parameter $\lambda^2=(E_cA_c/mgL)\,(8h/L)^3$ (see reference [1] for the notation); the finite oscillations are still governed only by the parameter λ^2 if the motion amplitude is non-dimensionalized with respect to the cable sag [6]. Therefore, any discussion of the influence of the various quantities appearing in the problem (E_c, A_c, h/L, L, m, H) on the dynamical behaviour should be developed within the ambit of this non-dimensionalization, even when the non-dimensionalization with respect to the cable length is preferable from a technical point of view. However, as different treatments used by different authors produce a slight discrepancy in the results obtained if approximations adopted are of the same level, a complete description of dynamical phenomenon of free in-plane oscillation for cables with various characteristics was given in the figures published in reference [6].

In this connection, even though the asymptotic solution is limited up to the second order, the trend observed in reference [1] (see Figure 4), namely that increasing the sag to span ratio results in a more pronounced hardening behaviour, does not fully account for what happens since, for example, with $h/L \geqslant 1/157$—which corresponds to $\lambda^2 \geqslant 4{\cdot}352$ with the values of the other parameters—the behaviour of the cable results is softening. Besides, it needs to be stressed that the results obtained for those particular values of E_c, A_c, q_z and L with the h/L ratio varying are actually valid for the whole set of cables with constant stiffness/weight ratio E_cA_c/q_zL [6, 9], since when non-dimensionalization with respect to span is referred to, the equation of free motion depends on only two parameters, one mechanical and the other geometrical.

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As previously observed, an asymptotic solution up to the second order furnishes a suitable description of the frequency-amplitude curves in a limited amplitude range [5] and in those cases as well—as for the cables studied in reference [1]—which exhibit monotonic hardening behaviour. This circumstance was observed in reference [3]; there the need to use a numerical procedure to obtain the frequency-amplitude curve of arches which are softening at low vibration amplitudes and hardening at high amplitudes was pointed out. On the contrary, in our opinion an asymptotic expansion can suitably describe this typical behaviour of cables if the analysis is carried out at an order higher than the second. This has been confirmed by the comparison illustrated in reference [6] between perturbational and numerical solutions.

Other points can be made regarding the description of the kinematics of a cable oscillating in the plane of static equilibrium. In equations of motion both longitudinal and vertical displacements u and w are involved. If only vertical oscillations are being studied, the longitudinal dynamical motion can be ignored; however, to put directly the longitudinal displacement equal to zero can be questionable. The $u(s, t)$ component can be eliminated by different procedures [4, 10, 6], giving just one differential equation in the variable $w(s, t)$, but in any case its effect is maintained and the greater the amplitude the greater is the result. The vertical stiffness of the cable increases in particular and this contributes to hardening behaviour. Which is the best procedure to eliminate the displacement u was discussed in reference [6], both by an analytical investigation and by studying the problem via a finite element model and a numerical temporal solution.

The vertical displacement variable w can be well described by the eigenfunction of the linearized problem furnished in reference [8]. On the other hand, the approximate function $\sin n\pi x$ used in reference [1] holds good for low values of λ^2 but not for $\lambda^2 > 4\pi^2$, which corresponds to values of h/L ratios greater than 1/44 for the values of mechanical and geometrical parameters considered in Figure 3 of reference [1]. In fact in these cases the first mode shows three waves.

As to the non-planar finite motion of a cable, it is well known that the in-plane and out-of-plane oscillations are coupled—a phenomenon which is not revealed by the linearized equations of motion. The study of this coupling phenomenon is often done by assuming one mode for every displacement variable which appears in the continuum dynamical equations. This is necessary to reduce the problem to two equations only in terms of the two modes involved in the coupling phenomenon. This is not possible if each displacement component is represented as a linear combination of the eigenfunctions [1], since in these conditions each equation would contain non-linear terms depending on the amplitudes of all the eigenfunctions of the problem.

The non-linear coupling terms between one in-plane mode and one out-of-plane are such that the in-plane oscillation can exist by itself while the out-of-plane oscillation is always coupled with the in-plane one, the amplitude of which is of a higher order if its initial value is zero [5]. This behaviour is what one would expect from a mechanical point of view and coincides with the well-known behaviour of the elastic pendulum [11]. Moreover, the analysis of equations of motion highlights two cases of internal resonance: (a) when the in-plane frequency ω_w is double and out-of-plane ω_v; (b) when $\omega_w = \omega_v$. The first case is peculiar to cables and does not occur with a taut string. This is just due to the non-linear terms associated with initial curvature and so it is a resonance of quadratic order. The second case is associated with cubic terms and the identity $\omega_v = \omega_w$ can exist only for a taut string. It is worth remembering these two cases of internal resonance for cables and their different orders as this strongly influences the amplitude of the region of resonance and the degree of the coupling phenomenon. This is much greater in the neighbourhood of $\omega_w = 2\omega_v$ than of $\omega_v = \omega_w$ [7].

In conclusion the paper by Al-Noury and Ali is of greater interest, but some aspects, summarized above, that are typical of non-linear dynamic behaviour of cables have been overlooked. The authors seem to have drawn on dynamic studies of taut strings and, in fact, they have considered preferably cables with characteristics similar to those of a taut string.

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