Appendix II.—Reference

8. Shears, M., "Static and Dynamic Behaviour of Guyed Masts," Structure and Materials Research, Report No. 68-6, Department of Civil Engineering, University of California, Berkeley, June, 1968.

Discussion by Angelo Luongo[5] and Giuseppe Rega[6]

The authors are congratulated for their interesting paper. Referring to results of their own work, the writers would like to make some comments regarding the proposed physical insight into the free vibration of parabolic cables and the presented approximate frequency spectrum.

One interesting circumstance pointed out by the authors is that the frequency increase occurring for symmetric modes of vibration with increasing relative stiffness parameter ρ is bounded up with mobilization of extensional rigidity of cable, the extent of which depends on the value of ρ and on the order of mode under consideration.

Such reinterpretation of Irvine's treatment (6) of cable free dynamics in terms of extensional resistance is contained already in Ref. 8, where the phenomenon of crossover is shown to be essentially a matter of kinematical compatibility. Indeed a description of this phenomenon is made in terms of a cable extensibility parameter $D_c = L/2EA$ and of a crossover extensibility for each mode, defined as $D_n^* = (8y_{max}/L)^3/24\,\rho_n^* q_v$. When $D_c = D_n^*$ there is a crossover point, when $D_c < D_n^*$ $(\rho > \rho_n^*)$ the frequency of the nth symmetric mode is greater while when $D_c > D_n^*$ $(\rho < \rho_n^*)$ the frequency of the nth antisymmetric mode is greater; D_n^* is thus the lowest extensibility the cable must have in order for inversion of the nth pair of modes to occur. By holding constant all cable parameters except E, the previous observations are restated by saying that only for E lower than a crossover value E_1^* the cable extensibility is sufficient for the symmetric configuration with one half-wave to occur, otherwise the first mode is an antisymmetric one. Besides, crossover is facilitated by an increase of L—that increases D_c and reduces D_n^*—which allows it to occur for higher values of E as well.

In the same paper, such reinterpretation is carried out to show how the phenomenon of crossover, being bound up with an overall extensibility parameter, i.e. the total cable elongation required by the one half-wave configuration, occurs provided there is some degree of extensibility in the system, whether distributed along the cable or lumped at the supports: in both cases kinematical conditions occur such that the first mode of the taut string can happen.

However, interpreting variation of ρ as variation of extensional rigidity of cable with constant y_{max}/L value has a conceptual limitation in the range of low ρ values, where the cable has very low extensional rigidity. In this range indeed the axial inertia forces should not be neglected since

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the frequencies of a longitudinal mode and of a transverse mode are comparable, entailing the dynamic behavior of cable cannot be described any more by the unique parameter ρ; so variation of ρ can be thought as variation of the sag ratio only.

Interesting results are also presented by the authors with regard to variation with ρ of $\Delta T_o/T_o$. Similar results were obtained by the writers (12) within a more restricted analysis for cables with $y_{max}/L = 1/12$ and 1/4 by using an inextensible model, which holds good at such sag ratio values; in particular, the reasonably expected result of greater values of $\Delta T_o/T_o$ for higher orders of mode was pointed out.

As regards the approximate spectra of natural frequencies, they can be surely useful in most practical purposes, though referring to the exact frequency curves does not seem complex in many cases. For instance, in Refs. 13 and 14 where nonlinear free vibrations of parabolic cables are studied, Eq. 8 and the corresponding axial displacement component are used in an assumed mode technique for analyzing the dependence of frequency vibration on the amplitude of the finite motion. In Ref. 14, in particular, the modification of the crossover phenomenon caused by such relationship is discussed, showing that, depending on the amplitude, an infinite number of crossover points between a symmetric and an antisymmetric mode occurs instead of the single one of the linear theory, so that at first vibration mode can be the symmetric one with three half-waves even with $\rho > \rho_1^* = \pi^2/3$.

Appendix.—References

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12. Luongo, A., and Rega, G., "An Inextensible Model for the Analysis of Free Vibrations of Suspended Cables," *Costruzioni Metalliche*, No. 3, 1980, pp. 140–152.
13. Hagedorn, P., and Schäfer, B., "On Non-linear Free Vibrations of an Elastic Cable," *International Journal of Non-Linear Mechanics*, Vol. 15, 1980, pp. 333–339.
14. Rega, G., Vestroni, F., and Benedettini, F., "Parametric Analysis of Large Amplitude Free Vibrations of a Suspended Cable," Istituto di Scienza delle Costruzioni, Università dell'Aquila, Report No. 54, L'Aquila, Italy, 1982 (to appear on *International Journal of Solids and Structures*).